

Automatic Control over the Cosmological Constant through Non-minimal Phantom and Quintessence

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A mechanism to control the cosmological constant through a scalar field non-minimally coupled to gravity is proposed. By utilizing non-minimal phantom or quintessence, the cosmological constant, which may be large originally, can be automatically driven to a value on the scale of the mass parameter in the phantom/quintessence potential $V(\phi)$. The reduction of a large cosmological constant involves the weakening of gravity that therefore may be much stronger initially. There exist the cases where originally gravity is on the TeV scale so that the hierarchy between gravity and three gauge interactions in the standard model of particle physics is bridged at the beginning. Although the cosmological constant can be automatically tuned or largely reduced under this mechanism, its energy density may still remain on the same order of magnitude as the original one. Thus, explaining the smallness of the observation-suggested cosmological constant energy density is still a difficult mission yet to be completed.

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Introduction. The existence of a small positive cosmological constant (CC) or, more conservatively, an upper limit to the CC is indicated by a variety of astrophysical observations [1, 2, 3, 4, 5, 6, 7]. This observation-suggested CC (upper limit) is on the scale $H_0 \sim 10^{-33}\text{eV}$, i.e. with the energy density on the scale $\sim 10^{-3}\text{eV}$. In contrast, a large CC from vacuum energy is expected in the framework of quantum field theory, whose energy density is on the scale of the quantum fluctuations under consideration. If this vacuum energy exists and does gravitate, even the quantum fluctuations on the micron scale can have ruined our universe in the early time, needless to say the Planck scale M_{Pl} or the supersymmetry breaking scale M_{SUSY} that may be the relevant scale for the vacuum energy. This contrast between the very large and the very small CC is a long-standing issue called “cosmological constant problem” [8]. For a review of possible approaches to solving this problem, see Refs. 9 and 10 and references therein.

The large CC from vacuum energy might be cancelled in a brute-force manner by finely tuning, for example, the size of the bare CC. As another example of fine tuning, in the Randall-Sundrum (RS) scenario [11] of extra dimension, by requiring a relation among the CC in the bulk and those on the branes, the effect of these CCs on the evolution of space-time can be cancelled, thereby allowing the existence of a static metric solution, even though all the CCs may be on a very large scale such as the Planck scale M_{Pl} .

Although such cancellation is artificial, unnatural and therefore not beautiful to the physicists full of the sense of beauty, no physical law forbids the creator from exploiting this brute-force cancellation to create a comfortable universe for human beings to reside in. Nevertheless, even accepting this not so lovely cancellation, one still

has difficulty of having a small CC, as described in the following.

In addition to the difficulty from the contrast between the very large and the very small CC mentioned above, the possible change of the vacuum energy during the phase transition associated with spontaneous symmetry breaking (SSB) makes this issue severe. Even if one obtains a small CC before the SSB phase transition with a delicate device, after the phase transition the CC energy density may drop for a certain amount on the scale of the phase transition, e.g. $\sim 300\text{GeV}$ for the electroweak symmetry breaking, thereby ruining the earlier (nearly) perfect cancellation. To have perfect cancellation after the phase transition(s), the creator must foresee all possible phase transitions and know the very details of the amount of the vacuum energy density change during each of them, as detailed as 10^{-3}eV at least. Then, the creator needs to make the CC cancellation before the phase transition imperfect, with the energy density deficit on the scale of the phase transition with the precision 10^{-3}eV or better. This fine tuning as a mission impossible for the creator of our universe plays a crucial part of the CC problem.

To overcome this obstacle, it will be perfect if the CC can be controlled automatically to a value one needs. That is, no matter how large the original CC and the CC changes at some later times were, the CC would eventually come to the required value. In the present article a mechanism to control the CC in such a way is explored. This mechanism is played by a scalar field, phantom (with negative kinetic energy) or quintessence (with ordinary positive kinetic energy), which is non-minimally coupled to gravity. As going to be presented, through this mechanism the original bare CC under control can be arbitrarily large and its sign can be either positive or negative. The sign of the resulting CC can be either positive or negative with the use of non-minimal phantom, while it is always negative with non-minimal quintessence [12]. As to the size of the resulting CC, stabilizing the

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CC around a moderate or a large value is achievable, which may be helpful to the construction of the models involving larger CC, such as the RS model. Nevertheless, controlling the CC to a small value, e.g. with the energy density on the scale of 10^{-3}eV as suggested by observations, is still a task yet to be done.

In this automatic CC control mechanism, the reduction of a large bare CC involves the weakening of gravity. Accordingly, the gravity before the performance of this mechanism to reduce the CC should be stronger. This may address the hierarchy problem regarding the contrast between gravity on the Planck scale, M_{Pl} , and three fundamental gauge interactions in the standard model of particle physics characterized by the scale $M_{\text{SM}} \sim \text{TeV}$, which is 16 orders of magnitude smaller than M_{Pl} . As going to be shown, there do exist the cases where originally the gravity is on the TeV scale, thereby bridging this hierarchy at the very beginning.

Analysis. Consider a real scalar field Φ which is non-minimally coupled to gravity, as described by the follow-

ing action with a power-law potential.

$$S = -\frac{1}{2\kappa} \int d^4x \sqrt{g} (\mathcal{R} + 2\Lambda_0) + \int d^4x \sqrt{g} \left[\frac{S_K}{2} g^{\mu\nu} (\partial_\mu \Phi) (\partial_\nu \Phi) - V_{\mathcal{R}}(\Phi) \right], \quad (1)$$

$$V_{\mathcal{R}}(\Phi) = \frac{1}{2} \xi \mathcal{R} \Phi^2 + V(\Phi), \quad (2)$$

$$V(\Phi) = S_V M^{4-n} \Phi^n, \quad M > 0. \quad (3)$$

Here κ is the gravitational constant characterizing the original strength of gravity, \mathcal{R} the Ricci scalar, Λ_0 the original bare cosmological constant, ξ the non-minimal coupling constant, and S_K and S_V denote the sign of the kinetic energy and that of the potential energy (for positive Φ^n), respectively. Accordingly, $S_K = +1$ corresponds to quintessence and $S_K = -1$ to phantom.

The scalar field equation and the Einstein equations corresponding to the above action are as follows.

$$0 = S_K \Phi^{;\alpha}_{;\alpha} + \partial_\Phi V_{\mathcal{R}}(\Phi) = S_K \Phi^{;\alpha}_{;\alpha} + [\xi \mathcal{R} \Phi + n S_V M^{4-n} \Phi^{n-1}], \quad (4)$$

$$G_{\mu\nu} = \Lambda' g_{\mu\nu} + \kappa_{\text{eff}} (T_{\mu\nu}^{\text{R}} + T_{\mu\nu}^{\text{M}}) + \kappa_{\text{eff}} [S_K (\partial_\mu \Phi) (\partial_\nu \Phi) - \mathcal{L}_\Phi^{(0)} g_{\mu\nu} + \xi (\Phi^2)_{;\mu;\nu} - \xi (\Phi^2)^{;\alpha}_{;\alpha} g_{\mu\nu}], \quad (5)$$

$$\mathcal{L}_\Phi^{(0)} = \frac{1}{2} S_K g^{\alpha\beta} (\partial_\alpha \Phi) (\partial_\beta \Phi) - V(\Phi), \quad (6)$$

where the semicolon ‘;’ denotes the covariant derivative, $T_{\mu\nu}^{\text{R}}$ and $T_{\mu\nu}^{\text{M}}$ are the energy contributions from radiation and (pressureless) matter, respectively, and the “modified cosmological constant” Λ' and the “effective gravitational constant” κ_{eff} are defined as follows.

$$\Lambda'(\Phi) \equiv \frac{\Lambda_0}{1 + \kappa \xi \Phi^2}, \quad (7)$$

$$\kappa_{\text{eff}}(\Phi) \equiv \frac{\kappa}{1 + \kappa \xi \Phi^2}. \quad (8)$$

For the present time $\kappa_{\text{eff}} = 8\pi G_N$, where G_N is the Newtonian gravitational constant. The contraction of Eq. (5) gives

$$\begin{aligned} \mathcal{R} &= -4\Lambda' - \kappa_{\text{eff}} [\rho_{\text{m}} + 4V - S_K \Phi^{;\alpha} \Phi_{;\alpha} - 3\xi (\Phi^2)^{;\alpha}_{;\alpha}] \\ &= -4\Lambda_{\text{eff}} - \kappa_{\text{eff}} \rho_{\text{m}} \quad \text{for constant } \Phi, \end{aligned} \quad (9)$$

where ρ_{m} is the matter energy density and the “effective cosmological constant” Λ_{eff} is defined as

$$\Lambda_{\text{eff}}(\Phi) \equiv \Lambda'(\Phi) + \kappa_{\text{eff}}(\Phi) V(\Phi) = \frac{\Lambda_0 + \kappa S_V M^{4-n} \Phi^n}{1 + \kappa \xi \Phi^2}. \quad (10)$$

Note that the radiation energy makes no contribution to the Ricci scalar \mathcal{R} .

In the following we investigate the structure of the non-minimal potential $V_{\mathcal{R}}(\Phi)$ — finding extrema and exploring their stability. The equation $\partial_\Phi V_{\mathcal{R}}(\Phi_m) = 0$ gives the location of extrema:

$$-\xi [4\Lambda_{\text{eff}}(\Phi_m) + \kappa_{\text{eff}}(\Phi_m) \rho_{\text{m}}] \Phi_m + n S_V M^{4-n} \Phi_m^{n-1} = 0, \quad (11)$$

or, equivalently,

$$\Phi_m [4\Lambda_0 + \kappa \rho_{\text{m}} + (4-n) \kappa S_V M^{4-n} \Phi_m^n - n S_V M^{4-n} \Phi_m^{n-2} / \xi] = 0. \quad (12)$$

For $n > 1$ there is an extremum at $\Phi_m = 0$. In addition, according to the above equation, there may exist another extremum at $\Phi_m = v \neq 0$, for which we have

$$\Lambda_{\text{eff}}(v) = \frac{1}{4\xi} n S_V M^{4-n} v^{n-2} - \frac{1}{4} \kappa_{\text{eff}}(v) \rho_{\text{m}}. \quad (13)$$

The model with $n = 2$ is particularly interesting and will be the focus in the rest of this article. For $n = 2$,

$$\Lambda_{\text{eff}}(v) = \frac{1}{2\xi} S_V M^2 - \frac{1}{4} \kappa_{\text{eff}}(v) \rho_{\text{m}}, \quad (14)$$

and accordingly the dependence of the resulting effective CC, $\Lambda_{\text{eff}}(v)$, on the original bare CC, Λ_0 , is only

through $\kappa_{\text{eff}}(v)$ appearing in the second term in the right-hand side. Furthermore, when the contribution from the scalar field potential dominates over the matter contribution, remarkably, the effective CC, Λ_{eff} , is independent of the original bare CC, Λ_0 , and is dictated simply by M (provided that ξ is of order unity):

$$\Lambda_{\text{eff}} \cong \frac{1}{2\xi} S_V M^2 \sim \pm M^2 \quad \text{as} \quad \rho_m \ll \left| \frac{M^2}{\kappa_{\text{eff}} \xi} \right|. \quad (15)$$

That is, in this case, even if the magnitude of the bare CC, Λ_0 , is very large originally, the scalar field Φ will evolve toward v at which the CC is balanced in such a way that the scale of the resulting effective CC, Λ_{eff} , is characterized by the mass parameter M in the potential, provided that $\Phi_m = v$ stands for a stable extremum (which will be investigated later). On the other hand, when the matter contribution dominates, the energy density associated with the effective CC follows the matter density with an opposite sign:

$$\rho_{\Lambda_{\text{eff}}} \equiv \Lambda_{\text{eff}}/\kappa_{\text{eff}} \cong -\frac{1}{4}\rho_m \quad \text{as} \quad \rho_m \gg \left| \frac{M^2}{\kappa_{\text{eff}} \xi} \right|. \quad (16)$$

Now we investigate the stability of extrema and explore the condition for the existence of a stable extremum at $\Phi_m = v \neq 0$, meanwhile fulfilling $\kappa_{\text{eff}}(v) = 8\pi G_N > 0$. The following is the relevant information for the $n = 2$ model.

$$\Phi_m = 0 \quad \text{or} \quad \Phi_m^2 = \frac{1}{\kappa \xi} - \frac{4\Lambda_0 + \kappa \rho_m}{2S_V \kappa M^2} \equiv v^2, \quad (17)$$

$$\kappa_{\text{eff}}(0) = \kappa, \quad \kappa_{\text{eff}}(v) = \frac{2S_V \kappa M^2}{4S_V M^2 - 4\xi \Lambda_0 - \kappa \xi \rho_m}, \quad (18)$$

$$\Lambda_{\text{eff}}(0) = \Lambda_0, \quad \Lambda_{\text{eff}}(v) = \frac{1}{2\xi} S_V M^2 - \frac{1}{4} \kappa_{\text{eff}}(v) \rho_m. \quad (19)$$

For $\Phi = \Phi_m + \delta\Phi$, the perturbation $\delta\Phi$ satisfies the following field equation.

$$S_K (\delta\Phi)^{;\alpha}_{;\alpha} - (4\xi \Lambda_0 + \kappa \xi \rho_m - 2S_V M^2) \delta\Phi = 0 \quad \text{as} \quad \Phi_m = 0, \quad (20)$$

$$S'_K (\delta\Phi)^{;\alpha}_{;\alpha} - 4S_V \kappa_{\text{eff}}(v) \xi M^2 v^2 \delta\Phi = 0 \quad \text{as} \quad \Phi_m = v, \quad (21)$$

where

$$S'_K \equiv S_K + 6\kappa_{\text{eff}}(v) \xi^2 v^2, \quad (22)$$

which is always positive for quintessence when $\kappa_{\text{eff}}(v) > 0$, but can be either positive or negative for phantom. For the stable extremum at $\Phi_m = v$ to exist in our universe where $\kappa_{\text{eff}}(v) = 8\pi G_N$, it is required that

$$\kappa_{\text{eff}}(v) > 0, \quad v^2 > 0, \quad S'_K S_V \xi < 0. \quad (23)$$

In terms of the new variables,

$$A_0 \equiv \Lambda_0 + \frac{1}{4} \kappa \rho_m, \quad A_M \equiv \frac{S_V M^2}{2\xi}, \quad y \equiv \frac{2(S_K + 3\xi)}{S'_K + 6\xi}, \quad (24)$$

the formulae for the quantities involved in the requirement in Eq. (23) can be rewritten as follows.

$$v^2 = \frac{1}{\kappa \xi} \left(1 - \frac{A_0}{A_M} \right), \quad (25)$$

$$\kappa_{\text{eff}}(v) = \frac{\kappa}{2 - A_0/A_M}, \quad (26)$$

$$S'_K = (S_K + 6\xi) \left(\frac{A_0/A_M - y}{A_0/A_M - 2} \right). \quad (27)$$

When the contribution from the matter density is negligible, A_0 represents the original bare CC that would be driven to the effective one of the size $\simeq A_M$. Accordingly, the ratio A_0/A_M appearing in the above equations represents the extent of the CC tuning or reduction that is considered huge in the CC hierarchy problem.

Results. In Table I, we exhaust all cases through exploring various ranges or values of S_K , S_V , ξ and $A_0/|A_M|$, while leaving the sign of κ unspecified. The cases where the requirement in Eq. (23) for the existence of a stable extremum with positive κ_{eff} is fulfilled are marked by the symbol “O”, and the other cases marked by “X”. These feasible cases marked by “O” are singled out and listed in Table II. The name of each feasible case is given in the first column “ID” that contains the information about the sign or the value of S_K , κ , S_V and ξ which are relevant quantities for specifying our model. The sign of the resulting effective CC and the extent of the automatic CC tuning in these feasible cases are presented in the columns A_M and $A_0/|A_M|$. The feasible cases marked by the star symbol “ \star ” are particularly interesting because the original bare CC under control in these cases can be arbitrarily large in its magnitude. Note that in the feasible quintessence cases the resulting effective CC is always negative, while in the case of phantom, with different settings, the resulting effective CC, as well as the original bare CC under control, can be either positive or negative. (For example, P_{++2}/P_{+-3} can drive an arbitrarily large negative/positive CC to a positive/negative value.)

In the limit $|A_0/A_M| \gg 1$, i.e., when the CC tuning involves significant CC reduction,

$$v^2 \simeq -(A_0/A_M) \cdot (\kappa \xi)^{-1}, \quad (28)$$

$$\kappa_{\text{eff}}(v) \simeq -(A_0/A_M)^{-1} \cdot \kappa \simeq (\xi v^2)^{-1}. \quad (29)$$

Requiring $\kappa_{\text{eff}} \sim M_{\text{Pl}}^{-2}$, one obtains

$$v^2 \sim M_{\text{Pl}}^2 / \xi, \quad (30)$$

$$\kappa \sim -(A_0/A_M) \cdot M_{\text{Pl}}^{-2}. \quad (31)$$

According to Eq. (30), for this automatic CC control mechanism to work for arbitrarily large bare CC, the non-minimal coupling constant ξ must be positive, which is exhibited in the \star cases. Eq. (31) indicates an important feature that through the process of the CC reduction under this mechanism, gravity is weakened by a factor A_M/A_0 .

TABLE I: All cases (including the feasible cases and others), presented in two tables: the left table for quintessence and the right for phantom. The left part of each table contains the information about the range of ξ and A_0 (in unit of $|A_M|$) that, as well as $\{S_K, S_V, \kappa\}$, are relevant quantities for specifying our model. In the right part the sign of the quantities involved in the requirement in Eq. (23) are presented, where $\pm\kappa$ denotes the sign which is the same as or opposite to that of κ . When the signs in the columns v^2 and $\kappa_{\text{eff}}(v)$ are the same and that in the column $-S'_K S_V \xi$ is positive, the extent of the automatic CC tuning indicated in the column $A_0/|A_M|$ can be achieved as long as the sign of κ is specified in the way such that v^2 and $\kappa_{\text{eff}}(v)$ are both positive. These feasible cases are marked by the symbol “O” in the last column, while the others marked by “X”. [The feasibility of the cases where $\xi = -1/6$ (for quintessence) or $1/6$ (for phantom) can also be read from this table, with the help of the following information: for quintessence, when $\xi = -1/6^\pm$, $y = \pm\infty$; for phantom, when $\xi = 1/6^\pm$, $y = \mp\infty$.]

Quintessence ($S_K = +1$)								Phantom ($S_K = -1$)									
$S_V = +1$								$S_V = +1$									
ξ	A_M	y	$A_0/ A_M $	S'_K	v^2	$\kappa_{\text{eff}}(v)$	$-S'_K S_V \xi$	ξ	A_M	y	$A_0/ A_M $	S'_K	v^2	$\kappa_{\text{eff}}(v)$	$-S'_K S_V \xi$		
$< -\frac{1}{6}$	-	< 1	< -2	-	$+\kappa$	$-\kappa$	-	X	< 0	-	$(1, 2)$	< -2	-	$+\kappa$	$-\kappa$	-	X
			$(-2, -1)$	+	$+\kappa$	$+\kappa$	+	O				$(-2, -y)$	+	$+\kappa$	$+\kappa$	+	O
			$(-1, -y)$	+	$-\kappa$	$+\kappa$	+	X				$(-y, -1)$	-	$+\kappa$	$+\kappa$	-	X
			$> -y$	-	$-\kappa$	$+\kappa$	-	X				> -1	-	$-\kappa$	$+\kappa$	-	X
$(-\frac{1}{6}, 0)$	-	> 2	$< -y$	+	$+\kappa$	$-\kappa$	+	X	$(0, \frac{1}{6})$	+	> 2	< 1	-	$+\kappa$	$+\kappa$	+	O
			$(-y, -2)$	-	$+\kappa$	$-\kappa$	-	X				$(1, 2)$	-	$-\kappa$	$+\kappa$	+	X
			$(-2, -1)$	+	$+\kappa$	$+\kappa$	+	O				$(2, y)$	+	$-\kappa$	$-\kappa$	-	X
			> -1	+	$-\kappa$	$+\kappa$	+	X				$> y$	-	$-\kappa$	$-\kappa$	+	O
> 0	+	$(1, 2)$	< 1	+	$+\kappa$	$+\kappa$	-	X	$> \frac{1}{6}$	+	< 1	$< y$	+	$+\kappa$	$+\kappa$	-	X
			$(1, y)$	+	$-\kappa$	$+\kappa$	-	X				$(y, 1)$	-	$+\kappa$	$+\kappa$	+	O
			$(y, 2)$	-	$-\kappa$	$+\kappa$	+	X				$(1, 2)$	-	$-\kappa$	$+\kappa$	+	X
			> 2	+	$-\kappa$	$-\kappa$	-	X				> 2	+	$-\kappa$	$-\kappa$	-	X
$S_V = -1$								$S_V = -1$									
ξ	A_M	y	$A_0/ A_M $	S'_K	v^2	$\kappa_{\text{eff}}(v)$	$-S'_K S_V \xi$	ξ	A_M	y	$A_0/ A_M $	S'_K	v^2	$\kappa_{\text{eff}}(v)$	$-S'_K S_V \xi$		
$< -\frac{1}{6}$	+	< 1	$< y$	-	$-\kappa$	$+\kappa$	+	X	< 0	+	$(1, 2)$	< 1	-	$-\kappa$	$+\kappa$	+	X
			$(y, 1)$	+	$-\kappa$	$+\kappa$	-	X				$(1, y)$	-	$+\kappa$	$+\kappa$	+	O
			$(1, 2)$	+	$+\kappa$	$+\kappa$	-	X				$(y, 2)$	+	$+\kappa$	$+\kappa$	-	X
			> 2	-	$+\kappa$	$-\kappa$	+	X				> 2	-	$+\kappa$	$-\kappa$	+	X
$(-\frac{1}{6}, 0)$	+	> 2	< 1	+	$-\kappa$	$+\kappa$	-	X	$(0, \frac{1}{6})$	-	> 2	$< -y$	-	$-\kappa$	$-\kappa$	-	X
			$(1, 2)$	+	$+\kappa$	$+\kappa$	-	X				$(-y, -2)$	+	$-\kappa$	$-\kappa$	+	O
			$(2, y)$	-	$+\kappa$	$-\kappa$	+	X				$(-2, -1)$	-	$-\kappa$	$+\kappa$	-	X
			$> y$	+	$+\kappa$	$-\kappa$	-	X				> -1	-	$+\kappa$	$+\kappa$	-	X
> 0	-	$(1, 2)$	< -2	+	$-\kappa$	$-\kappa$	+	O	$> \frac{1}{6}$	-	< 1	< -2	+	$-\kappa$	$-\kappa$	+	O
			$(-2, -y)$	-	$-\kappa$	$+\kappa$	-	X				$(-2, -1)$	-	$-\kappa$	$+\kappa$	-	X
			$(-y, -1)$	+	$-\kappa$	$+\kappa$	+	X				$(-1, -y)$	-	$+\kappa$	$+\kappa$	-	X
			> -1	+	$+\kappa$	$+\kappa$	+	O				$> -y$	+	$+\kappa$	$+\kappa$	+	O

TABLE II: Feasible cases. The name of each feasible case is given in the first column “ID”, where “P” and “Q” mean phantom and quintessence respectively and the information about the sign or the range of the essential quantities (for specifying a model), κ , V and ξ , are presented in the subscript in order. The sign of the resulting effective CC and the extent of the automatic CC tuning in these feasible cases are shown in the columns A_M and $A_0/|A_M|$. The cases where the original bare CC under control can be arbitrarily large in its magnitude are particularly interesting and marked by the star symbol “★”.

Quintessence ($S_K = +1$)					Phantom ($S_K = -1$)						
ID	κ	V	ξ	A_M $A_0/ A_M $	ID	κ	V	ξ	A_M $A_0/ A_M $	(remark)	
Q $_{++-}$	+	+	-	-	$(-2, -1)$	P $_{++1}$	+	+	< 0	-	$(-2, -y)$ $1 < y < 2$
*Q $_{++-}$	+	+	-	-	> -1	*P $_{++2}$	+	+	$(0, \frac{1}{6}]$	+	< 1
*Q $_{--+}$	-	-	+	-	< -2	P $_{++3}$	+	+	$> \frac{1}{6}$	+	$(y, 1)$ $y < 1$
						P $_{+-1}$	+	+	< 0	+	$(1, y)$ $1 < y < 2$
						*P $_{+-3}$	+	+	$> \frac{1}{6}$	-	$> -y$ $y < 1$
						*P $_{-+2}$	-	+	$(0, \frac{1}{6})$	+	$> y$ $y > 2$
						P $_{--2}$	-	-	$(0, \frac{1}{6})$	-	$(-y, -2)$ $y > 2$
						*P $_{--3}$	-	-	$\geq \frac{1}{6}$	-	< -2 $y < 1$

TABLE III: Interesting cases regarding the energy scales of A_0 (Λ_0), A_M (Λ_{eff}) and κ (the original strength of gravity).

$A_0^{1/2}$	$A_M^{1/2}$	$\kappa^{-1/2}$
	$A_0^{1/2}$	M_{Pl}
	$10^{-3} A_0^{1/2}$	M_{GUT}^a
	$10^{-16} A_0^{1/2}$	TeV^b
	A_0/M_{Pl}	$A_0^{1/2c}$
(1) (all Planck-scale)		
M_{Pl}	M_{Pl}^d	M_{Pl}
(2) (TeV gravity) ^b		
M_{Pl}	TeV	TeV^b
M_{GUT}	GeV	TeV^b
TeV	10^{-4}eV	TeV^{bc}
(3) ($\Lambda_{\text{eff}} \sim H_0^2$) ^e		
TeV	H_0^e	10^{-17}eV^f
M_{Pl}	H_0^e	H_0^f

^a $M_{\text{GUT}} \sim 10^{16}\text{GeV}$.

^bTeV-scale gravity originally.

^cThe same as the scale of the original bare CC.

^dPlanck-scale effective CC as required in the RS scenario.

^eThe current CC scale suggested by observations, $H_0 \sim 10^{-33}\text{eV}$.

^fExtremely strong gravity originally.

The change of the gravitational interaction strength involved in the CC tuning under this mechanism is a general feature. Regarding the strength of gravity and the scale of the CC, several interesting cases are demonstrated in Table III, and the discussions about the examples therein are as follows.

Case (1): All are around the Planck scale, benefiting the RS scenario where it is required to control the effective CCs on the branes and that in the bulk to some fixed values around the Planck scale.

Case (2): The original strength of gravity is on the TeV scale, and accordingly the hierarchy between gravity and three gauge interactions in the standard model of particle physics is bridged at the very beginning. The third example in this category is particularly interesting, where both gravity and the CC are on the TeV scale originally. This TeV-scale gravity eventually becomes the present Planck-scale one through the process of reducing

an originally TeV-scale CC to the scale of 10^{-4}eV . Note that the resulting effective CC is barely affected by the later CC change(s) as long as the scale the CC change is much smaller than TeV. That is, this CC control is stable against the CC change up to the TeV scale.

Case (3): For tuning a large CC to the H_0 -scale value suggested by observations, the gravity must be very strong originally, which leads to a fine tuning of the original strength of gravity.

Summary. It has been a task for a long time to explain the smallness of the CC against the huge contribution from the quantum fluctuations of vacuum and the possible significant change during the SSB phase transition. In the present article a mechanism to automatically control the CC via non-minimally coupled phantom and quintessence is proposed. While the quintessence can only direct the CC to a negative value, the phantom can generate an effective CC of some required size, either positive or negative, from an arbitrarily large bare CC. With this mechanism, for the RS scenario controlling the CCs on the branes and that in the bulk to some fixed values around the Planck scale can be achieved. In addition, controlling the CC to a value on the 10^{-4}eV scale or above, such as TeV, is also achievable. In the case involving significant CC reduction, gravity is weakened and accordingly should be stronger originally. There exist the cases where the gravity is on the TeV scale originally. This is particularly interesting because in these cases gravity and three gauge interactions in the standard model of particle physics can be on the equal footing, i.e. with no hierarchy between them, at the very beginning. Furthermore, in one of these cases, the original bare CC is also on the TeV scale that accordingly plays a particularly fundamental role in the beginning. Later the four fundamental interactions split into gravity and other three gauge interactions with the hierarchy of the 16 orders of magnitude, as accompanied by the reduction of the CC from the TeV scale to the 10^{-4}eV scale. Although the CC can be automatically controlled under this mechanism to a large extent, generating an effective CC on the observation-suggested H_0 scale is still a difficult mission yet to be completed.

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